Grade 6 Quantifying probability

6.SP.4	
Demonstrate an understanding of probability by • identifying all possible outcomes of a probability experiment • differentiating between experimental and theoretical probability • determining the theoretical probability of outcomes in a probability experiment • determining the experimental probability of outcomes in a probability experiment • comparing experimental results with the theoretical probability for an experiment.	1. List the possible outcomes of a probability experiment, such as • tossing a coin • rolling a die with a given number of sides • spinning a spinner with a given number of sectors 2. Determine the theoretical probability of an outcome occurring for a probability experiment. 3. Predict the probability of a given outcome occurring for a probability experiment by using theoretical probability. 4. Conduct a probability experiment, with or without technology, and compare the experimental results to the theoretical probability. 5. Explain that as the number of trials in a probability experiment increases, the experimental probability approaches theoretical probability of a particular outcome. 6. Distinguish between theoretical probability and experimental probability and explain the differences.

Clarification of the outcome:

- ✦ The outcome concerns the first time that students attach a number to probability (quantify probability). In previous grades they have been attaching language only (e.g. a spinner is more likely to land on red than blue).
- ✦ Theoretical probability requires a model of some kind. [e.g.: A coin has two sides and it is equally likely to land on either side. the probability of landing HEADS is 1/2.). Experimental probability requires data. [e.g. Pull a number out of a hat 10 times. The number 4 was pulled three times. The probability of pulling a '4' is 3/10.]

Required close-to-at-hand prior knowledge:

- ✤ Understand the part of a set/group meaning of fraction.
- ✤ Understand the connection between fractions, decimals, and percent.
- ✤ Understand what a probability experiment is (grade 5 outcome)

SET SCENE stage

Bring in newspapers and other reading material that contain probability situations stated as a percent (e. g. the chance of precipitation today is 40%) or as a fraction.

The problem task to present to students:

Ask students to examine the material and to find and record two different situations that concern chance/luck where percent or a fraction is used to describe chance/luck.

Comments:

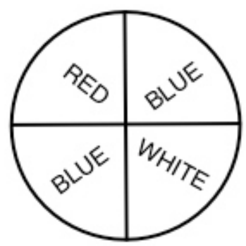
The purpose of the task is to expose students to situations that concern probability expressed as a number.

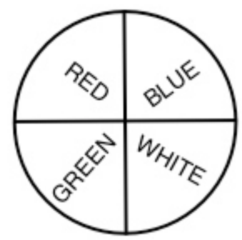
DEVELOP stage

Activity 1: Revisits SET SCENE and addresses achievement indicator 1.

Ask selected students to present situations they recorded about luck/chance. Discuss what they recorded to ensure students realize that a number can be attached to luck/chance to indicate how likely something will happen. Tell them that what follows will strengthen and deepen their understanding about probability. Define probability for them (concerns luck/chance).

- ✦ Show students a coin (e.g. a quarter). Ask them the ways it can land (expect: heads or tails). Tell them that each way of landing is known as an outcome when talking about probability.
- ✦ Show students a normal 6-sided die. Ask them what outcomes are possible when throwing the die (expect: six; 1, 2, 3, . . .6).
- ✦ Show students two different spinners (see examples). Ask them what outcomes are possible when spinning it.



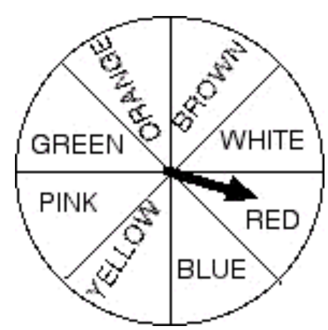


NOTE:

One of the spinners MUST have a duplicate colour. Students need to understand that a spinner stopping at the BLUE sections (for example) is two outcomes, not one outcome.

Activity 2: Addresses no achievement indicators (introduces experimental probability).

Organize students into pairs and give each pair the same spinner, one that is divided into identical segments of a different colour (see example). Pick a winning colour. Ask students to spin the spinner 20 times and record the number of times the winning colour occurs (RED for this example).



Ask for the results (the data) and discuss why they are not the same. Ensure students realize luck/chance behaves that way (i.e. there is no guarantee about what will happen).

Tell students that they were doing a probability experiment. Ask them how the chances of the winning colour happening might be represented with a number. Discuss their suggestions. Tell them that this is experimental probability.

Ensure students realize that experimental probability is represented by the fraction:

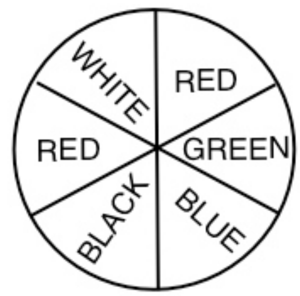
$$\frac{\text{number of wins}}{\text{number of tries}}$$

[*For this example, if RED occurred 3 times in 20 tries/trials, the experimental probability of RED occurring is 3/20 (or 15%)*.]

Activity 3: Addresses achievement indicators 1, 2, 4, and 6.

Revisit the spinner from activity 2.

- ✦ Ask students to list the possible outcomes of the spinner (for the example, there are 8 outcomes). Ask students to list the number of times the winning outcome (the winning colour) occurs on the spinner (for the example, once). Ask them what should be the probability of the winning colour happening. Ensure they realize it is the number of times the winning colour is on the spinner divided by the number of sections on the spinner (for the example spinner, 1/8). Tell them that this is theoretical probability. Compare the experimental probabilities from activity 2 to the theoretical probability. Ensure students realize they may or may not be the same value.
- ✦ Provide a different spinner, one that has the winning colour occurring twice on the spinner (RED in this example). Tell them the winning colour is the one that occurs twice. Ask students to determine the theoretical probability of the winning colour occurring (RED here). Discuss their solutions (for the example it would be 2/6).



- ✦ Organize students into pairs and give each pair the spinner indicated above. Ask students to spin the spinner 20 times and record the number of times the winning colour occurs (RED for this example). Ask them to determine the experimental probability.
- ✦ Compare the experimental probabilities from the spinner above to the theoretical probability. Discuss why the two values are not always the same. [Theoretical probability concerns what SHOULD HAPPEN in the long run. Experimental probability concerns what ACTUALLY HAPPENS.]

Activity 4: Addresses achievement indicators 1, 2, 4, 5, and 6.

- ✦ Organize students into pairs and provide each pair with a normal coin (e. g. a loonie). Ask them to determine the theoretical probability of the coin landing with HEADS showing as a fraction and as a percent. Ensure that they understand that the correct response is 1/2 and 50%.
- ✦ Ask each pair to toss the coin 20 times, recording the number of times HEADS occurs. Ask each pair to determine the experimental probability. Ask for and discuss the results. Ask them to compare the theoretical probability of 1/2 to the pair's experimental probability. Combine all the data from the pairs. Ask students to determine the experimental probability of the coin landing on HEADS for the combined data. They may notice that the theoretical probability and the experimental probability are closer in value for the combined data than for most of the separate data.
- ✦ Discuss that as the number of trials/tries in a probability experiment increases, the experimental probability approaches theoretical probability.

Activity 5: Addresses achievement indicators 1, 2, 4, 5, and 6 & practice.

Organize students into pairs and give each pair a 6-sided die that has the following numbers on it: **1, 2, 2, 3, 4, 5**.

- ✦ Ask students to express the theoretical probability of getting a 2. Ensure that they understand that the probability is 2/6 (or 1/3). Ask them to throw the die 24 times and determine the experimental probability.
- ✦ Ask them how the experimental and theoretical probabilities would compare if we combined the results from all the pairs of students.
- ✦ Combine the data and compare the experimental and theoretical probabilities of the combined data to uncombined data. Discuss the results. Ensure students realize that as the number of trials/tries in a probability experiment increases, the experimental probability approaches theoretical probability.

Activity 6: Addresses achievement indicators 1, 2, 3, 4, and 6.

- ✦ Organize students into pairs. One student in each pair receives a 4-sided die having the numbers 1 to 4 on it. The other person receives a normal 6-sided die. The students play the game 'Get 3'. Each student throws his/her die. If a '3' is showing then that student receives a point. If both dice show a '3' each student receives a point. The winner is the player reaching 4 points first. Ask for the results (which player won most often).
- ✦ Ask students to determine the theoretical probability of throwing a '3' for the 4-sided die and the 6-sided die. Ensure that students understand that, for the 4-sided die, the chance of getting a '3' is 1 out of 4 and that, for the 6-sided die, the chance of getting a '3' is 1 out of 6. Ask whether the game is fair. Ensure they realize that the person throwing the 4-side die has a better chance of winning because 1/3 is greater than 1/6.

Activity 7: Addresses achievement indicators 1, 2, 3, 4, and 6.

- ✦ Ask students to define experimental probability and theoretical probability. Ensure they generalize theoretical probability as:

$$\frac{\text{Number of ways to win}}{\text{Total number of outcomes}}$$

- ✦ Remind students about the probability situations that they found in the reading materials. Discuss the situations of:
  - whether experimental or theoretical probability and why
  - what outcomes might be involved
  - how data might have been collected
  - how the experimental/theoretical probabilities might have been determined

Activity 8: Addresses achievement indicators 1, 2, 3, 4, 5, and 6 & practice.

Prepare sufficient bags containing pieces of paper of the same size. The number of pieces in a bag must be a multiple of 10 (e. g. 10, 20, 30, etc.). All the papers in each bag are blank except for one (or two or three, . . .) that has the word, 'WINNER' written on it. [Put in sufficient pieces of paper with the word WINNER on them so that the theoretical probability of pulling a piece of paper with the word WINNER on it is 1/10 for each bag. For example, if there are 10 pieces of paper in the bag, only one of them will have the word WINNER on it. The rest of the pieces will be blank. For example, if there are 20 pieces of paper in the bag, two of them will have the word WINNER on them. The rest of the pieces will be blank, and so on.]

Organize students into pairs and give each pair a bag. Each pair pulls one piece of paper out of the bag and records if it was a WINNER. The paper is returned to the bag. Repeat 10 times. Ask each group to determine the experimental probability for pulling WINNER from its data.

Ask each pair to examine the contents of their bag and determine the theoretical probability of pulling out a piece of paper with the word WINNER on it. Discuss the results. Ensure that each group determines a theoretical probability that is equivalent to 1/10. Ask them to compare the theoretical probability to the experimental probability. Discuss why the two probabilities are likely different and what would happen if they pulled a paper out of the bag 10 000 times.

Activity 9: Addresses achievement indicators 1, 2, and 3, & practice.

Provide students with the following table of information. Tell them that it provides information on all the marbles in a bag.

Size of marble	Number of blue marbles	Number of red marbles
small	5	3
medium	8	10
large	3	4

Ask students to answer the following questions. Ask for and discuss their responses.

1. How many marbles in the bag? ____________
2. How many blue marbles? _____________
3. How many medium marbles? _____________

A marble is picked out of the bag, without looking into the bag.

4. What is the theoretical probability of the marble being blue? _____________
5. What is the theoretical probability of the marble being red? ______________
6. What is the theoretical probability of the marble being small? ____________
7. What is the theoretical probability of the marble being large and red? ____________

Activity 10: Assessment of teaching.

Ask students to describe a situation where the theoretical probability of doing something is 1/5 and where the experimental probability turns out to be 3/10. [For example: I have a bag with 5 marbles in it. One of the marbles in the bag is blue. The theoretical probability of pulling out a blue marble is 1/5. I pulled a marble out of the bag 10 times and returned it each time. Three of those times the marble was blue. The experimental probability of pulling out a blue marble is 3/10.]

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

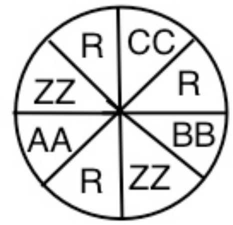
Two examples of partially well-designed worksheets follow.

Each worksheet contains a sampling of question types. More questions of each type are needed for a well-designed worksheet.

The MAINTAIN stage follows the sample worksheets.

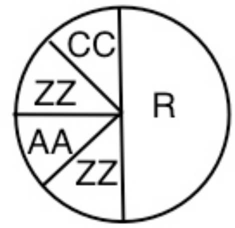
Question 1.

What is the theoretical probability of the spinner stopping on R?



Question 2.

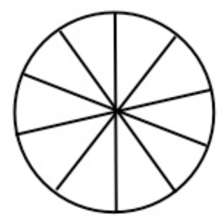
What is the theoretical probability of the spinner stopping on R?



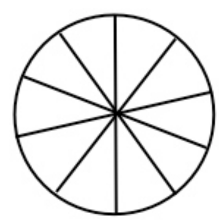
Question 3.

All of the sections of the spinner are equal in area. Write the letters A, B, and C on the spinner so that the spinner has:

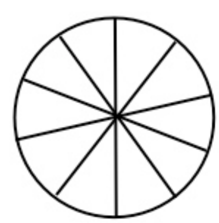
h. the theoretical probability of 3/10 for stopping on A.



i. the theoretical probability of 9/10 for stopping on B.



j. the theoretical probability of 1/2 for stopping on C.



Question 4.

A 6-sided die has the numbers, 0, 0, 1, 1, 6, 9, on it.

What is the theoretical probability of the die showing ‘0’ on top when it is thrown?

Question 5.

In a probability experiment, names are pulled out of a hat. After 30 pulls, the name ‘JOE’ was pulled out 7 times. What is the experimental probability of pulling out the name ‘JOE’?

Question 6.

The table shows age information about the boys and girls in a grade 5 class.

Age	Number of Boys	Number of Girls
10	2	1
11	5	8
12	6	6

1. How many students in the class? ________________
2. How many boys? ______________
3. How many 11-year old students? ______________

A student’s name is picked out of a hat. The student makes the morning announcement.

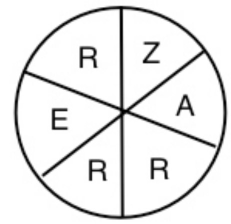
4. What is the theoretical probability of the student being a girl? __________
5. What is the theoretical probability of the student being a boy? __________
6. What is the theoretical probability of the student being a 12-year old girl? _____
7. What is the theoretical probability of the student being a 10-year old boy? _____

MAINTAIN stage

Mini-task example

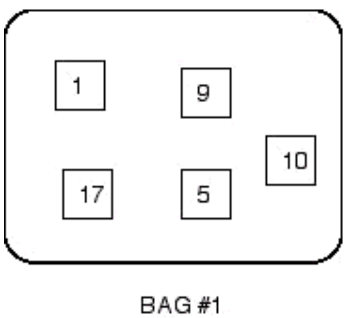
Every so often:

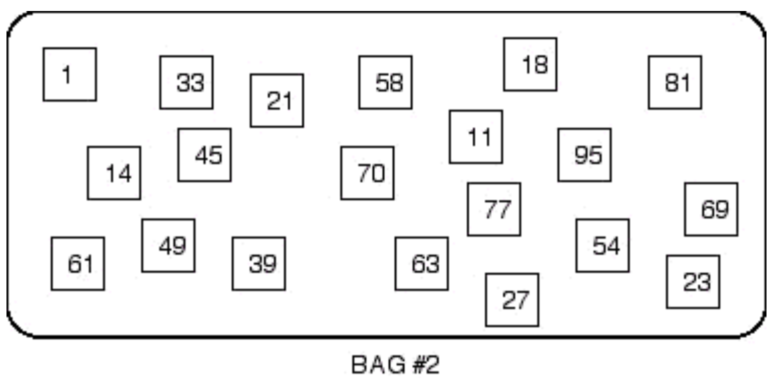
- Present a situation that involves theoretical probability (see spinner example). Ask students to determine the theoretical probability.



Rich-task example

There are two bags on a table at a carnival booth. Bag #1 contains five identical small cubes. Bag #2 contains 20 identical small cubes. While the cubes are identical in size, they have different numbers on them. See the diagrams below for the contents of the bags.





To play the game at the booth, you must first choose a bag (bag #1 or bag #2) from which to pull out a cube. After choosing the bag, you then pull one cube out of it. You win a prize if the cube has a prime number on it. Which bag would you choose? Explain why, using theoretical probability, expressing it as a fraction and as a percent. Test your decision by designing an experiment that allows you to compare the two bags, using experimental probability.

Comments

This is a rich-task because it is a complex problem that also is integrated with converting fractions to percents.